THERMAL PERFORMANCE AND SIZING OF MOVING BED HEAT EXCHANGERS

Pedro Isaza
University of Toronto
Toronto, Ontario, Canada

W. David Warnica
University of Toronto
Toronto, Ontario, Canada

Markus Bussmann
University of Toronto
Toronto, Ontario, Canada

ABSTRACT

A novel thermal performance relation is presented for a moving bed heat exchanger (MBHE) and its application is demonstrated via a sample rating calculation. Unlike conventional fluid-fluid heat exchangers, where effectiveness relations are a function of the heat capacity ratio, \( C \), and the number of thermal transfer units, \( NTU \), the MBHE expression also depends on the Biot number, \( Bi \), due to heat conduction in the solids. The effectiveness curve for an MBHE operating under the special condition \( C = 0 \) is presented, along with its application to a proposed system for solar central receiver plants [1].

INTRODUCTION

The use of granular solids as a means of storage for concentrated solar power has begun to receive increasing attention. Ceramics and natural stones offer simplicity and low cost, while effectively storing heat due to their high density, thermal conductivity and specific heat capacity [1]. However, transfer of energy from these materials requires the effective design of solid-to-fluid heat exchangers. A promising category of these are MBHEs, where flowing granular solids indirectly exchange energy with a moving fluid.

MBHEs are versatile because they can accommodate different flow arrangements (counter-current, parallel, or cross flow), while requiring minimal driving energy. However, a proper formulation and solution of the governing energy equations has yet to be presented, and limited amount of information is available for thermal performance and sizing.

In this work, novel performance relations are presented in the form of effectiveness curves, obtained by analytically solving the governing energy equations for a parallel plate MBHE subject to a constant temperature fluid. A rating application is then demonstrated for the system presented by Baumann and Zunf [1]. This work is unique because it begins to create a platform for thermal analysis of MBHEs, presenting effectiveness curves arising from novel analytical solutions to the heat transfer problem. The application of the curves on an existing system is demonstrated, with the aim of guiding design engineers.

GOVERNING EQUATIONS

Under many conditions of interest, moving solids can be described as a “single component” continuum [2] and an overall heat transfer coefficient, \( U_o \), linking the solid and fluid domains in an MBHE, can account for wall/particle contact, conduction through the wall, and convection into the fluid. Assuming steady conditions and constant properties, the governing energy equations have been solved analytically for solids moving at a constant velocity subject to heat conduction in the lateral direction (i.e., towards the fluid domain) and convection in the axial direction. Solutions for different flow arrangements (with respect to fluid direction) are obtained through separation of variables or Laplace transforms, and their final form is a series, similar to those of transient heat conduction problems. Effectiveness relations can then be obtained for thermal performance analysis.

Effectiveness, \( \varepsilon \), is defined as the ratio of actual and maximum heat transfer rate. The actual heat rate is the energy lost or gained by the fluid or solid, while the maximum rate is calculated from the largest temperature differential and the minimum capacity rate in the system, \( C = \dot{m} \cdot C_p \) (fluid or solids) [3]. For an MBHE, effectiveness is defined as follows:

\[
\varepsilon = \frac{\sigma_{\vartheta_w - \vartheta_{st}}}{(t_{f1} - t_{f0})} = \frac{1}{C} \frac{(t_{f1} - t_{f0})}{(t_{f1} - t_{f0})} \quad \text{for } C \leq 1
\]  

(1)

Like fluid-to-fluid relations, effectiveness depends on the heat capacity ratio, \( C \), and number of transfer units (NTU) defined as follows:

\[
C = \frac{\dot{m}_s \cdot C_p}{\dot{m}_f \cdot C_{pf}}
\]  

(2)

\[
NTU = \frac{U_o \cdot A_{st}}{\dot{m}_s \cdot C_p}
\]  

(3)

\( \varepsilon \) will also depend on the Biot number due to heat conduction in the solids:

\[
Bi = \frac{U_o \cdot \vartheta_{fw}}{k_s}
\]  

(4)
An important subcategory of the solutions takes place when the fluid maintains constant temperature. This occurs in phase change applications where \( C = 0 \). Many MBHEs operate in this regime, including those examined in solar collection systems [1]. Figure 1 below, depicts the effectiveness curves obtained for \( C = 0 \) as a function of NTU and Bi.

![Fig 1: Effectiveness vs. NTU and Bi for C=0](image)

Figure 1 it shows that as the Biot number approaches zero, the solution tends to the standard fluid-fluid effectiveness equation [3]. This is expected given that \( Bi \rightarrow 0 \) implies that the solids conduction resistance becomes negligible (with all other properties being constant). Under this condition, equal temperatures are predicted at all solids cross-sections with the exception of distributions within the contact resistance lumped in the overall heat transfer coefficient. As the Biot number increases, by virtue of increasing solids conduction resistance, the effectiveness relation departs from the fluid-fluid solution. Conduction in the solids becomes an important mechanism in the transport process, affecting the amount of energy exchanged. For example at an NTU of 1, \( \varepsilon \) is 0.632 for \( Bi = 0 \) compared to 0.103 for \( Bi = 100 \).

**APPLICATION:**

To demonstrate the application of Fig. 1, consider the MBHE geometry proposed by Baumann and Zunft [1] for quartz sand in their solar application. Outlet temperatures are not reported, so this exercise is a preliminary test of Fig. 1 for predicting the thermal performance of an actual MBHE. Introducing a geometric change, the shell and tube system is converted to parallel plates, with a new effective width and height, but equal depth. Constant solids velocity is assumed. An effective conductivity, \( k_s = 0.24 \frac{W}{mK} \), and contact resistance, \( R_c = 0.0067 \frac{m^2K}{W} \), for quartz sand is obtained by fitting the data of Niegsch et al. [4] with the model. The original and transformed geometries are summarized in Table 1.

Assuming that the estimated material properties remain constant under the new experimental conditions, a Biot number of 10.88 is found. Effectiveness and average solids outlet temperatures are predicted using Fig. 1 for the NTU’s estimated for the original system [1], where solid velocities range from 1-5 mm/s. For all tests, a constant fluid temperature of 250°C is reported along with a solids entrance temperature of 800°C. Table 2 summarizes the results.

**CONCLUSIONS AND NEXT STEPS**

This work introduces first-of-kind effectiveness curves for an MBHE, operating under constant fluid temperature conditions. Their application and use is demonstrated through thermal rating of an existing system in the literature for a solar collection process. Upcoming work will focus on presenting the analytical solutions and effectiveness plots for different MBHE arrangements (parallel, counter flow and cross-flow) and more general conditions (i.e., variable fluid temperature).

**REFERENCES**