

# Volume Averaging Towards a One-Equation Model for a Moving Bed Heat Exchanger

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## ABSTRACT

This paper presents the method of volume averaging for the microscopic energy equations of a moving bed heat exchanger. Macroscopic expressions, when both the interstitial fluid and solids are in motion, are developed and mathematical constraints are identified. A one-equation model is then constructed describing energy transport in the material. Major mathematical assumptions associated with this novel development are clearly stated and identified. This analysis demonstrates the implications associated with assuming a one-equation model, for describing energy transport in flowing granular materials.

## 1 INTRODUCTION

Many industrial processes hinge on the controlled heating and cooling of granular solids. In the food industry, controlled heat sterilization of powder products is particularly important when dealing with temperature sensitive peptides [1]. In the petrochemical industry, heating of granular solids affects the recovery from oil shale [2] while in nickel production, temperature control ensures efficient deposition from gases [3]. Effective heat transport in particulates also influences the emission abatement efficiency of  $SO_x$  and  $NO_x$  in certain processes [4]. Very recently, solids have been proposed as a heat storage medium for concentrated solar radiation [5].

An important heat exchanger used in the above applications is the moving bed heat exchanger (MBHE). In these systems, moving particulates consisting of an interstitial fluid and a solid phase flow on one side of the exchanger while indirectly exchanging energy with a heating or cooling fluid. MBHE types include both parallel-plate and shell and tube, while flow arrangements may be counter-current, parallel, or cross-flow. This versatility drives their low investment cost, energy consumption and maintenance requirements [6].

A complete understanding of the physics of heat trans-

fer in MBHEs has yet to be attained. This was a challenge in the area of packed beds (PBs) until a rigorous mathematical approach known as volume averaging (VA) was adopted [7]. At its core, VA acts as a transformation which upscales microscopic equations to produce macroscopic counterparts associated with representative elementary volumes (REV). This procedure of spatial smoothing is in its infancy with respect to MBHEs, where work by Pivem and de Lemos [6] stands as an early demonstration of the analysis. Unlike PBs, convection of transport properties also occurs due to solid flow in MBHEs. This additional mode of transport increases the complexity of the equations to be averaged.

To date, much of the work regarding volume averaging for moving bed heat exchangers has focused on the development of thermal non-equilibrium models. However, the mathematical constraints and validity of thermal equilibrium have yet to be explored to the same level of detail presented for PBs ([7], [8]). This exercise is overdue as pivotal research in MBHEs relies on this assumption ([9], [10], [11]).

The purpose of the work is to present the volume averaging of the microscopic energy equations when solids are in motion. Steps required to produce a one energy equation model for an MBHE are demonstrated, while mathematical constraints are identified along with important assumptions. Investigations of this kind are critical, given that analytical solutions can be conceived for describing MBHEs operating in the realm of the one-equation model. A complete analysis of this type would produce correlations for thermal rating and sizing.

## 2 GOVERNING ENERGY EQUATIONS IN MBHEs

Moving beds differ from packed beds in that the solids are in motion. A simplistic diagram depicting the general conditions of interest (at an REV level) is shown

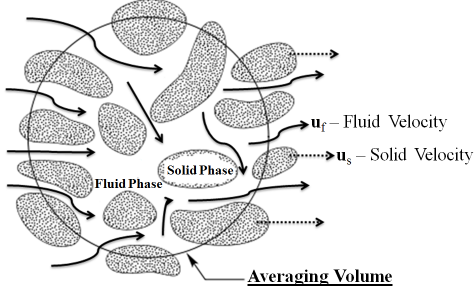


Figure 1: REV for a fluid-solid system in an MBHE

in Figure 1.

Assuming constant thermo-physical properties (heat capacity,  $C_p$ , density,  $\rho$ , and thermal conductivity,  $k$ ), negligible radiation and conversion of mechanical to thermal energy, the governing equations for an MBHE can be formulated as follows:

#### Fluid Phase Energy Equation

$$(\rho C_p)_f \frac{\partial T_f}{\partial t} + (\rho C_p)_f \nabla \cdot (\mathbf{u}_f T_f) = \nabla \cdot (k_f \nabla T_f) \quad (1)$$

#### Solid Phase Energy Equation

$$(\rho C_p)_s \frac{\partial T_s}{\partial t} + (\rho C_p)_s \nabla \cdot (\mathbf{u}_s T_s) = \nabla \cdot (k_s \nabla T_s) \quad (2)$$

At a microscopic level, the problem is complemented by several boundary conditions. Analogous to the work of Whitaker for packed beds ([7], Sec. 2.1), the following expressions apply to an MBHE:

#### BC#1 - Temperature Continuity at the Solid-Fluid Interface

$$T_f = T_s \quad \text{at } \mathbf{A}_{fs} \quad (3)$$

#### BC#2 - Flux Balance at the Solid-Fluid Interface

$$-\mathbf{n}_{fs} \cdot k_f \nabla T_f = -\mathbf{n}_{fs} \cdot k_s \nabla T_s \quad \text{at } \mathbf{A}_{fs} \quad (4)$$

#### BC#3 - Impermeable and Moving Fluid Interface

$$\rho_f (\mathbf{u}_f - \mathbf{w}) \cdot \mathbf{n}_{fs} = 0 \quad \text{at } \mathbf{A}_{fs} \quad (5)$$

#### BC#4 - Impermeable and Moving Solid Interface

$$\rho_s (\mathbf{u}_s - \mathbf{w}) \cdot \mathbf{n}_{sf} = 0 \quad \text{at } \mathbf{A}_{sf} \quad (6)$$

where  $\mathbf{w} \cdot \mathbf{n}_{sf}$  is the speed of displacement of the fluid-solid interface.

### 3 VOLUME AVERAGING FUNDAMENTALS

Volume averaging begins by associating every point in a global domain with REV's considered invariant with time and space [7]. A position vector  $\mathbf{x}$  locates the REV centroids, while a relative position vector  $\mathbf{y}$  defines any point in the phase of interest within the REV. Subscripts on  $\mathbf{y}$  identify the phase being specified ([7], Sec. 1.2). Volume integration follows and the superficial average of a property is defined as:

$$\langle \psi_\beta \rangle = \frac{1}{V} \int_{V_\beta(t)} \psi_\beta dV \quad (7)$$

where the  $\beta$  subscript details the phase of interest,  $\psi_\beta$  the microscopic property in the  $\beta$  phase,  $V_\beta(t)$  the volume of  $\beta$  in the REV,  $V$  the volume of the REV, and  $\langle \psi_\beta \rangle$  the *superficial* property at the REV centroid.

Intrinsic and superficial properties are connected through the following relation:

$$\langle \psi_\beta \rangle = \varepsilon_\beta \langle \psi_\beta \rangle^\beta \quad (8)$$

where  $\varepsilon_\beta$  is the volume fraction of the  $\beta$ -phase in the REV, and  $\langle \psi_\beta \rangle^\beta$  is the intrinsic property.

Two supplementary theorems are required to completely volume average the microscopic energy equations. The first is spatial averaging, a three dimensional representation of the Leibniz rule ([7], [12], [13], [14], [15], [16], [17]) given by:

$$\langle \nabla \psi_\beta \rangle = \nabla \langle \psi_\beta \rangle + \frac{1}{V} \int_{A_{\beta\alpha}(t)} \mathbf{n}_{\beta\alpha} \psi_\beta dA \quad (9)$$

where  $\mathbf{n}_{\beta\alpha}$  is the normal unit vector pointing from the  $\beta$  to the  $\alpha$  phase (second phase in the REV), and  $A_{\beta\alpha}(t)$  is the interfacial area between phases.

The second theorem is the general transport theorem:

$$\left\langle \frac{\partial \psi_\beta}{\partial t} \right\rangle = \frac{\partial \langle \psi_\beta \rangle}{\partial t} - \frac{1}{V} \int_{A_{\beta\alpha}(t)} \psi_\beta \mathbf{w} \cdot \mathbf{n}_{\beta\alpha} dA \quad (10)$$

where  $\mathbf{w} \cdot \mathbf{n}_{\beta\alpha}$  is the speed of displacement of the interface.

### 4 VOLUME AVERAGING OF MBHE ENERGY EQUATIONS

Since Eqs. 1 and 2 have equivalent forms, only the development of the fluid equation is presented. Volume averaging Eq. 1 yields:

$$(\rho C_p)_f \frac{1}{V} \int_{V_f(t)} \left[ \frac{\partial T_f}{\partial t} + \nabla \cdot (\mathbf{u}_f T_f) \right] dV = \frac{1}{V} \int_{V_f(t)} [\nabla \cdot (k_f \nabla T_f)] dV \quad (11)$$

## 4.1 Transient Energy Term

To interchange the order of integration and differentiation in the transient term in Eq. 11, the general transport theorem is applied leading to

$$\left\langle \frac{\partial T_f}{\partial t} \right\rangle = \frac{\partial \langle T_f \rangle}{\partial t} - \frac{1}{V} \int_{A_{fs}(t)} T_f \mathbf{w} \cdot \mathbf{n}_{fs} dA \quad (12)$$

Introducing Eq. 8 results in:

$$\left\langle \frac{\partial T_f}{\partial t} \right\rangle = \frac{\partial (\epsilon_f \langle T_f \rangle^f)}{\partial t} - \frac{1}{V} \int_{A_{fs}(t)} T_f \mathbf{w} \cdot \mathbf{n}_{fs} dA \quad (13)$$

## 4.2 Convective Energy Term

Applying the vector form of the spatial averaging theorem to the convective term in Eq. 11, yields the following:

$$\langle \nabla \cdot (\mathbf{u}_f T_f) \rangle = \nabla \cdot \langle \mathbf{u}_f T_f \rangle + \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \cdot (\mathbf{u}_f T_f) dA \quad (14)$$

To analyze this expression further, a spatial decomposition of variables is proposed. Intrinsic and point properties are linked via spatial deviation terms as detailed by Gray [18]. The following correlations are put forward:

$$T_f = \langle T_f \rangle^f + \tilde{T}_f \quad (15)$$

$$\mathbf{u}_f = \langle \mathbf{u}_f \rangle^f + \tilde{\mathbf{u}}_f \quad (16)$$

where  $\tilde{T}_f$  and  $\tilde{\mathbf{u}}_f$  are the fluid spatial deviations of temperature and velocity respectively.

Applying these expressions to the first term in Eq. 14 yields:

$$\nabla \cdot \langle \mathbf{u}_f T_f \rangle = \nabla \cdot \langle \langle T_f \rangle^f \langle \mathbf{u}_f \rangle^f + \langle T_f \rangle^f \tilde{\mathbf{u}}_f + \tilde{T}_f \langle \mathbf{u}_f \rangle^f + \tilde{T}_f \tilde{\mathbf{u}}_f \rangle \quad (17)$$

Following an analysis equivalent to that of Whitaker ([7], Sec. 3.2.3), where several Taylor series expansions are applied, the following simplified expression results:

$$\langle \mathbf{u}_f T_f \rangle = \epsilon_f \langle T_f \rangle^f \langle \mathbf{u}_f \rangle^f + \langle \tilde{T}_f \tilde{\mathbf{u}}_f \rangle \quad (18)$$

Substituting Eq. 18 into 14 provides:

$$\langle \nabla \cdot (\mathbf{u}_f T_f) \rangle = \nabla \cdot (\epsilon_f \langle T_f \rangle^f \langle \mathbf{u}_f \rangle^f) + \nabla \cdot \langle \tilde{T}_f \tilde{\mathbf{u}}_f \rangle + \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \cdot (\mathbf{u}_f T_f) dA \quad (19)$$

where  $\nabla \cdot (\epsilon_f \langle T_f \rangle^f \langle \mathbf{u}_f \rangle^f)$  and  $\nabla \cdot \langle \tilde{T}_f \tilde{\mathbf{u}}_f \rangle$  are the traditional convective and dispersive flux terms arising during volume averaging.

## 4.3 Conductive Energy Term

Applying the vector form of the spatial averaging theorem to the conduction term in Eq. 11, we obtain:

$$\langle \nabla \cdot (k_f \nabla T_f) \rangle = \nabla \cdot \langle k_f \nabla T_f \rangle + \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \cdot k_f \nabla T_f dA \quad (20)$$

Implementing the theorem a second time, allows the first term on the RHS of the expression to be written as:

$$\nabla \cdot \langle k_f \nabla T_f \rangle = \nabla \cdot (k_f \nabla \langle T_f \rangle) + \nabla \cdot \left[ k_f \left( \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} T_f dA \right) \right] \quad (21)$$

Collecting everything together (and substituting Eq. 8 as needed), a volume averaged conduction term of the following form is found:

$$\begin{aligned} \langle \nabla \cdot (k_f \nabla T_f) \rangle &= \nabla \cdot (k_f \nabla (\epsilon_f \langle T_f \rangle^f)) \\ &+ \nabla \cdot \left[ k_f \left( \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} T_f dA \right) \right] \\ &+ \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \cdot k_f \nabla T_f dA \end{aligned} \quad (22)$$

## 4.4 Volume Averaged Energy Balances

Equations 13, 19 and 22 can now be substituted into 11 to obtain the volume averaged equation for the fluid phase. Carrying out the substitution and applying the boundary condition for fluid impermeability, given by Eq. 5, we obtain:

### Volume Averaged Fluid Phase Energy Equation

$$\begin{aligned} (\rho C_p)_f \left[ \frac{\partial (\epsilon_f \langle T_f \rangle^f)}{\partial t} + \nabla \cdot (\epsilon_f \langle T_f \rangle^f \langle \mathbf{u}_f \rangle^f) + \nabla \cdot \langle \tilde{T}_f \tilde{\mathbf{u}}_f \rangle \right] \\ = \nabla \cdot \left[ k_f \left( \nabla (\epsilon_f \langle T_f \rangle^f) + \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} T_f dA \right) \right] \\ + \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \cdot k_f \nabla T_f dA \end{aligned} \quad (23)$$

An equivalent analysis of the solids microscopic energy balance reveals an analogous expression to Eq. 23 of the form:

### Volume Averaged Solid Phase Energy Equation

$$\begin{aligned} (\rho C_p)_s \left[ \frac{\partial (\epsilon_s \langle T_s \rangle^s)}{\partial t} + \nabla \cdot (\epsilon_s \langle T_s \rangle^s \langle \mathbf{u}_s \rangle^s) + \nabla \cdot \langle \tilde{T}_s \tilde{\mathbf{u}}_s \rangle \right] \\ = \nabla \cdot \left[ k_s \left( \nabla (\epsilon_s \langle T_s \rangle^s) + \frac{1}{V} \int_{A_{sf}(t)} \mathbf{n}_{sf} T_s dA \right) \right] \\ + \frac{1}{V} \int_{A_{sf}(t)} \mathbf{n}_{sf} \cdot k_s \nabla T_s dA \end{aligned} \quad (24)$$

## 5 ONE-EQUATION MODEL FOR MBHES

Expressions 23 and 24 are now added together to move towards a one-equation model. The following expression results after their addition, and the implementation of the flux BC given by Eq. 4:

$$\begin{aligned}
& (\rho C_p)_f \left[ \frac{\partial(\epsilon_f \langle T_f \rangle^f)}{\partial t} + \nabla \cdot (\epsilon_f \langle T_f \rangle^f \langle \mathbf{u}_f \rangle^f) + \nabla \cdot \langle \tilde{T}_f \tilde{\mathbf{u}}_f \rangle \right] \\
& + (\rho C_p)_s \left[ \frac{\partial(\epsilon_s \langle T_s \rangle^s)}{\partial t} + \nabla \cdot (\epsilon_s \langle T_s \rangle^s \langle \mathbf{u}_s \rangle^s) + \nabla \cdot \langle \tilde{T}_s \tilde{\mathbf{u}}_s \rangle \right] \\
& = \nabla \cdot \left[ k_f \left( \nabla (\epsilon_f \langle T_f \rangle^f) + \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} T_f dA \right) \right] \\
& + \nabla \cdot \left[ k_s \left( \nabla (\epsilon_s \langle T_s \rangle^s) + \frac{1}{V} \int_{A_{sf}(t)} \mathbf{n}_{sf} T_s dA \right) \right]
\end{aligned} \tag{25}$$

### 5.1 Constant Solid Velocity

We consider now the simplest case where the solids are **assumed** to move at constant speed. From the spatial decomposition relation for velocity, analogous to Eq. 16 but for the solids, the deviation term is assumed negligible and we find:

$$\mathbf{u}_s = \langle \mathbf{u}_s \rangle^s = \text{constant}; \quad \tilde{\mathbf{u}}_s = 0 \tag{26}$$

### 5.2 Constant Porosity

An important supplementary assumption is that of constant porosity:

$$\epsilon_f = \text{constant}; \quad \epsilon_s = \text{constant} \tag{27}$$

This condition is reasonable in the free flowing domain of the bulk material, but less so near walls and/or obstructions. A special analysis, of the style presented by Ochoa-Tapia and Whitaker [19], would be required to circumvent this limitation.

### 5.3 Incompressible Fluid

Constant thermo-physical properties were assumed for both phases during the development of the microscopic energy equations, and therefore the continuity equation for the fluid has the following form:

$$\nabla \cdot (\mathbf{u}_f) = 0 \tag{28}$$

Applying the spatial averaging theorem we find:

$$0 = \nabla \cdot \langle \mathbf{u}_f \rangle + \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \cdot \mathbf{u}_f dA \tag{29}$$

From BCs#3 and #4, a supplementary relation can be constructed at the fluid-solid interface of the following form:

$$\mathbf{u}_f \cdot \mathbf{n}_{fs} = \mathbf{u}_s \cdot \mathbf{n}_{fs} \quad \text{at } \mathbf{A}_{fs} \tag{30}$$

As demonstrated by Whitaker ([7], Sec. 1.3.1), the zeroth order superficial spatial moment is the following:

$$\nabla \epsilon_f = -\frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} dA \tag{31}$$

Given that constant porosity is assumed, the spatial moment simplifies to:

$$0 = \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} dA \tag{32}$$

Applying the above relation in Eq. 29, alongside the constant solids velocity assumption, we find the following volume averaged continuity equation:

$$\nabla \cdot \langle \mathbf{u}_f \rangle^f = 0 \tag{33}$$

### 5.4 Local Thermal Equilibrium

A final assumption is that of local thermal equilibrium (LTE). The validity of this assumption can be explored mathematically (see transient examples in packed beds by Quintard and Whitaker [8]); however, this is left as future work. Local thermal equilibrium implies equal intrinsic temperature in both phases (at an REV level) as follows:

$$\langle T_f \rangle^f = \langle T_s \rangle^s = \langle T \rangle \tag{34}$$

### 5.5 One Equation Model

Applying the equations (and associated assumptions) detailed in sections 5.1-5.4 allows for Eq. 25 to be written as:

$$\begin{aligned}
& \underbrace{[\epsilon_f (\rho C_p)_f + \epsilon_s (\rho C_p)_s]}_{\text{Accumulation}} \frac{\partial \langle T \rangle}{\partial t} \\
& + \underbrace{[\epsilon_f (\rho C_p)_f \langle \mathbf{u}_f \rangle^f + \epsilon_s (\rho C_p)_s \mathbf{u}_s]}_{\text{Convection}} \cdot \nabla \langle T \rangle + \\
& \underbrace{(\rho C_p)_f \nabla \cdot \langle \tilde{T}_f \tilde{\mathbf{u}}_f \rangle}_{\text{Thermal Dispersion}} = \nabla \cdot \left[ \underbrace{(k_f \epsilon_f + k_s \epsilon_s)}_{\text{Conduction}} \nabla \langle T \rangle \right] \\
& + \nabla \cdot \left[ \underbrace{(k_f - k_s)}_{\text{Tortuosity}} \left( \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} T_f dA \right) \right]
\end{aligned} \tag{35}$$

where the temperature continuity boundary condition, given by Eq. 3, is used to simplify the area integral terms.

As a final step, the integral within the tortuosity term is evaluated further. Applying a spatial decomposition of variables, we find:

$$\begin{aligned} \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} T_f dA &= \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \langle T \rangle dA \\ &+ \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \tilde{T}_f dA \end{aligned} \quad (36)$$

Following an analysis equivalent to that of Whitaker ([7], Sec. 1.3), where a Taylor series is used to expand the volume averaged temperature, the following simplified expression results:

$$\frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} T_f dA = \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \tilde{T}_f dA \quad (37)$$

subject to the mathematical constraints:

$$(k_f \boldsymbol{\varepsilon}_f + k_s \boldsymbol{\varepsilon}_s) \nabla \langle T \rangle \gg (k_f - k_s) \langle T \rangle \nabla \boldsymbol{\varepsilon}_f \quad (38)$$

$$(k_f \boldsymbol{\varepsilon}_f + k_s \boldsymbol{\varepsilon}_s) \nabla \langle T \rangle \gg (k_f - k_s) \nabla \langle \mathbf{y}_f \rangle \cdot \nabla \langle T \rangle \quad (39)$$

$$(k_f \boldsymbol{\varepsilon}_f + k_s \boldsymbol{\varepsilon}_s) \nabla \langle T \rangle \gg (k_f - k_s) \frac{1}{2} \nabla \langle \mathbf{y}_f \mathbf{y}_f \rangle : \nabla \nabla \langle T \rangle \quad (40)$$

Clearly the first inequality is met by virtue of the constant porosity assumption. The latter two, however, need to be kept in mind and explored further for moving solid systems. This analysis would follow a procedure similar to that presented by Whitaker for packed beds [7], and entails order of magnitude estimates.

Substituting Eq. 37 into 35, yields a one energy equation model for moving solids:

$$\begin{aligned} &\underbrace{[\boldsymbol{\varepsilon}_f (\rho C_p)_f + \boldsymbol{\varepsilon}_s (\rho C_p)_s]}_{\text{Accumulation}} \frac{\partial \langle T \rangle}{\partial t} \\ &+ \underbrace{[\boldsymbol{\varepsilon}_f (\rho C_p)_f \langle \mathbf{u}_f \rangle^f + \boldsymbol{\varepsilon}_s (\rho C_p)_s \mathbf{u}_s]}_{\text{Convection}} \cdot \nabla \langle T \rangle \\ &+ \underbrace{(\rho C_p)_f \nabla \cdot \langle \tilde{T}_f \tilde{\mathbf{u}}_f \rangle}_{\text{Thermal Dispersion}} = \nabla \cdot \left[ \underbrace{(k_f \boldsymbol{\varepsilon}_f + k_s \boldsymbol{\varepsilon}_s) \nabla \langle T \rangle}_{\text{Conduction}} \right] \\ &+ \nabla \cdot \left[ \underbrace{(k_f - k_s) \left( \frac{1}{V} \int_{A_{fs}(t)} \mathbf{n}_{fs} \tilde{T}_f dA \right)}_{\text{Tortuosity}} \right] \end{aligned} \quad (41)$$

A closure problem remains to be developed in order to link volume averaged and spatial deviation properties. This entails proposing a REV micro-structure, and the analysis of the resulting boundary value problem (see [7]). However, the objectives of the current document are now met. A governing one-equation model for an MBHE has been formulated (for simplified conditions,) and the major mathematical constraints in the development have been identified.

## 6 CONCLUSIONS

This work presents the implementation of volume averaging techniques to formulate the macroscopic energy balances for an MBHE. The mathematical analysis required to obtain a one energy equation is demonstrated. Inequalities, or constraints, associated with these conditions are identified. Subsequent work will examine the development of analytical solutions for describing MBHEs operating in the realm of this one energy equation.

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